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DETERMINATION OF THE INTENSITY OF A FREE VORTEX SHEET WITHIN THE FRAMEWORK OF LIFTING SURFACE THEORY

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UDC 533.69

A free vortex sheet leaving the edge of a small-span wing exerts substantial influence on its lifting properties. The problem of the flow around a finite span wing is a potential stream reduces in lifting surface theory to singular integrodifferential equations with a free surface. The method of discrete vortices turns out to be effective for the solution of problems on the flow around a wing with a free surface. A discrete vortex scheme is proposed in [1] for a finite span thin wing in which the intensity of the vortex line congruent with the wing leading (including side) edge is assumed equal to the intensity of an attached line vortex that goes over into a free vortex on the wing independently of the wing planform. However, as comparison between the numerical computations performed [2] and experimental results shows, the intensity of a free vortex sheet, exactly as its configuration, depends on the wing planform and the degree of wing roundoff and deflection. In the case of thin wings with plane middle surface, the principal factor governing the convergent vortex sheet is the wing planform. A discrete vortex sheet is proposed in [2-4] for a finite-span wing, where the dependence of the intensity of the free vortex sheet on the wing geometry is taken into account by the introduction of a parameter $0 \leqslant K \leqslant 1$. The value $K = 0$ on the wing corresponds to the regime of unseparated flow around a wing, and $K = 1$ to the model used in [1]. The magnitude of the parameter $0 \leqslant K \leqslant 1$ for a specific sweep angle was established by comparing computed and experimental results.

Convergent conditions for finite-span wings are formulated in this paper that permit determination of the dependence of the free vortex sheet on the local wing sweep angle in explicit form by starting from the line vortex conservation conditions in a potential flow. Results of a computation of the total aerodynamic characteristics of rectangular and triangular wings of different spans are compared with existing experimental data obtained in wind tunnels.

In conformity with vortical lifting surface theory, the vortex surface attached to the wing trailing edge goes smoothly over into a free vortex surface; the line vortex intensity is here conserved. On the wing leading (side) edges, the nature of the runoff of the vortical layers from the lifting surface is determined by their interaction with the free stream. According to available experiments on bubble visualization in flow channels and oil—soot visualization of streamline tracks on wing surfaces in a wind tunnel, the streamline (line vortex) pattern in the flow over a triangular thin plate at an angle of attack is shown schematically in Fig. 1a, where the free vortex surface convergent with the leading edge is displayed on the left side of the wing, and streamlines on the upper wing surface, on the right. The leading edge is the envelope of surface stream lines approaching the wing and the line vortices convergent with the wing. The attached line vortex (vortex tube) is divided into two components upon approaching the edge L and touching it; a line that has a direction in agreement with the tangent to the wing edge, and remains connected to the wing and a line that converges with the wing surface while becoming a free line vortex (Fig. 1b). The binormal velocity component from the free stream side hinders runoff of the thin vortical layers at the leading edge. As numerous experiments show, the intensity of the vortex sheet coincident with the leading edge in thin wings around which there is flow at an angle of attack will grow as the sweep angle of the edge increases.

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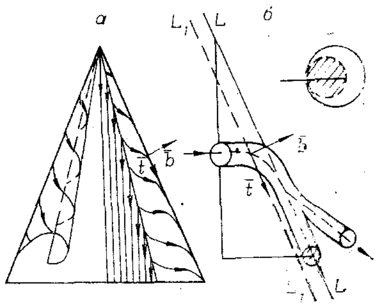

Fig. 1

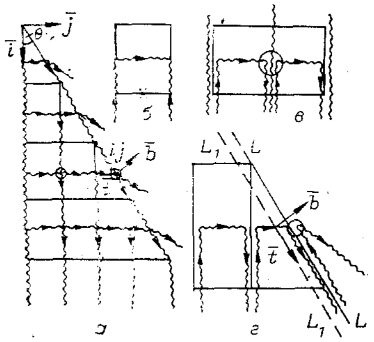

Fig. 2

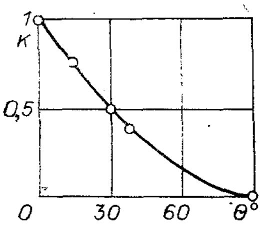

Fig. 3

The problem of separation flow around a wing lifting surface by an incompressible subsonic stream can be represented as a problem of the flow around attached and free vortex surfaces. Starting from the Green's formula [5]

$$\int_V \operatorname{rot} \mathbf{u} dV = \oint_S [\mathbf{n} \times \mathbf{u}]\, dS,$$

it can be shown that the vortex density vector on a vortical surface S is represented as a discontinuity in the tangential velocities on this surface

$$\boldsymbol{\gamma} = [\mathbf{n} \times [\mathbf{u}_\tau\,]], \tag{1}$$

where $\mathbf{n}$ is the unit vector normal to the surface S; $[\mathbf{u}_\tau] = \mathbf{u}_{\tau w} - \mathbf{u}_{\tau l}$; $\mathbf{u}_{\tau w}$, $\mathbf{u}_{\tau l}$ are the tangential velocity components on the windward and leeward sides of the surface, respectively. The discontinuity in the normal velocity component is $[\mathbf{u}_n]_S = 0$.

For a certain line L on the vortex surface S the tangential velocity vector $\mathbf{u}_\tau$ can be decomposed into binormal and tangential components, and in conformity with this and with (1) the vorticity vector can be represented in the form

$$\boldsymbol{\gamma}_L = \mathbf{b}\gamma_{bL} + \mathbf{t}\gamma_{tL} = \mathbf{b}\,[u_t]_L - \mathbf{t}\,[u_b]_L, \tag{2}$$

where $\mathbf{b}$, $\mathbf{t}$ are unit vectors of the binormal and tangent at the point of the line L being considered, $[u_b]_L$, $[u_t]_L$ are the binormal and tangential components of the discontinuities in the velocity moduli ($\mathbf{u}_\tau = \mathbf{b}u_b + \mathbf{t}u_t = \mathbf{u}_b + \mathbf{u}_t$).

If the line L coincides with the wing leading edge, then it is a special branching line vortex. Let us select a line L_1 located a small distance ε equidistantly from the edge L on the wing (see Fig. 1b) and having identical tangent and binormal directions with the edge L at corresponding points, and we decompose the vortex density γ_{L_1} on this line vortex into binormal and tangential components in conformity with (2). The assertion is propounded that bifurcation of the attached line vortex into two components occurs on the wing edge: The attached component corresponding to the tangential component of the vortex density vector to the line L_1 remains on the wing while the remaining part becomes a free line vortex.

According to (2), the tangential component of the vorticity vector is determined by the discontinuity in the binormal velocity component on the line $L_1 \to L$ of the vortical surface S. In particular, if the discontinuity in the binormal velocity component to the edge is zero at the point under consideration, then the tangential component γ_{tL} of the vorticity vector equals zero and a line vortex of intensity equal to the intensity of the attached line vortex approaching the edge at this point converges with the wing — the case K = 1 is realized. If the discontinuity of the tangential velocity component to the edge is zero at the point under consideration, then the binormal component γ_{bL} of the vorticity vector is zero there and the line vortex does not converge with the wing, which corresponds to unseparated flow around the edge (K = 0). Let us note that an a priori determination of the quantities $[u_b]_L$, $[u_t]_L$ (therefore, the intensities of the convergent vortex sheet) is not possible in the general case of a nonplanar wing of arbitrary planform since these quantities are only found after having solved the problem of the flow around a wing by the selected scheme with a given convergent vortex sheet intensity.

In a discrete vortex scheme for a wing, when the direction of the attached line vortices is fixed, even when approaching the wing edges, the magnitude of the tangential component of the attached line vortex on the wing edge can be determined and by using the propounded assertion about the nature of the bifurcation of the vorticity vector on the edge, the intensity of the convergent line vortex can thereby be determined.

Later, all the discussion will be for the example of a plane triangular wing. The dependences obtained for a line vortex on the straight leading edge of a delta wing are applicable to the curvilinear edge of a plane wing.

The wing is partitioned into elements of a rectangular mesh (Fig. 2a). A rectangular Π-shaped vortex whose transverse component connects the middle of the rectangular mesh sides parallel to the wing axis of symmetry is set in correspondence with each element of the lifting surface, while the longitudinal components go to infinity (Fig. 2b). The Π- and Γ-shaped vortices with branches going to infinity are set in correspondence with the boundary triangle of the mesh that contains the leading edge L. One of the branches of the Γ-shaped vortex is the free line vortex [3, 6]. Addition of the longitudinal components of the Π-shaped vortices and the components of the vortices of the boundary triangles directed along the edge L results in the construction of a mesh of line vortices on the wing (fragments of the formation of line vortex nodes of Fig. 2a, marked by circles, are isolated in circles in Fig. 2c, d).

The attached vortex filament $\mathbf{\Gamma}_{ijL} = \mathbf{j}\Gamma_{ijL}$ perpendicular to the free stream (the unit vector $\mathbf{i}$ is directed parallel to the wing axis, while the unit vector $\mathbf{j}$ is perpendicular to the axis, Fig. 2a) approaches the point ij of the edge L in the boundary cell of the wing. The component of this vortex filament that is directed along the wing edge

$$\mathbf{\Gamma}_{ijtL} = \mathbf{t}\mathbf{\Gamma}_{ijL} \sin\theta, \tag{3}$$

where θ is the slope of the leading edge to the wing axis of symmetry, remains the attached line vortex according to the assertion made above, the other component becomes the free line vortex. The intensity of the free vortex filament is determined on the line L_1 equidistant from the edge L. The attached vortex filament $\mathbf{\Gamma}_{ijL}$ is still not bifurcated on the line L_1. The transverse section dS of the attached complete vortex tube is shown at the point of intersection of its axis with the line L_1 in the upper part of Fig. 1b. The shaded part of this section dS_l corresponds to the vortex filament component that remains attached, while the unshaded part dS_f is the component that becomes the free line vortex on L. The attached vortex filament is characterized by the following relationships prior to bifurcation:

$$\mathbf{\Gamma}_{ijL} = \mathbf{j}\Gamma_{ijL} = \mathbf{j}(\gamma_{ijL}dS) = \mathbf{j}\gamma_{ijL}(dS_l + dS_f) = \mathbf{j}(\Gamma_{ijtL} + \Gamma_{ijcL}). \tag{4}$$

Approaching the edge L the vortex filament $\mathbf{\Gamma}_{ijL}$ is curved and bifurcated into two.

The curvature and bifurcation domain is not fixed on the schematic discrete vortex pattern (see Fig. 2a). This domain on the wing between the lines L_1 and L (Figs. 1b and 2d) is represented in Fig. 2a by the wing edge ($L_1 \to L$). The decomposition of the vortex filament approaching the wing into binormal and tangential components is performed at the point of its intersection with the edge L. The intensity of the branch corresponding to the tangential component and remaining the attached branch is determined according to (3) and (4) by the formula

$$\Gamma_{ijtL} = \Gamma_{ijL} \sin\theta. \tag{5}$$

Starting from the properties of conservation of the vortex tube intensity, the intensity of that part of the total vortex tube that becomes the free vortex filament is found in conformity with (4) and (5) by the formula

$$\Gamma_{ijcL} = \Gamma_{ijL} - \Gamma_{ijtL} = \Gamma_{ijL}(1 - \sin\theta). \tag{6}$$

Therefore, the parameter K, introduced earlier in [2-4], and governing the intensity of the convergent vortex sheet, depends on the slope of the edge to the wing axis of symmetry

$$K = 1 - \sin\theta. \tag{7}$$

This dependence is shown in Fig. 3.

If the wing leading edge is directed perpendicularly to the free stream, then $\theta = \pi/2$ and the intensity of the vortex sheet convergent with the edge is zero (K = 0), and the free stream velocity is directed directly opposite to the stream overflow velocity from the windward to the leeward sides.

If the edge is complanar to the free stream velocity, then θ = 0 and the intensity of the the convergent vortex sheet equals the vortical intensity on the wing in the neighborhood of the leading edge (K = 1). In this case there is no free stream velocity component hindering the stream overflow from the windward to the leeward side.

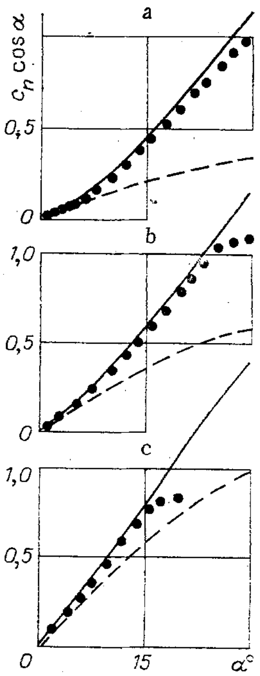

Fig. 4

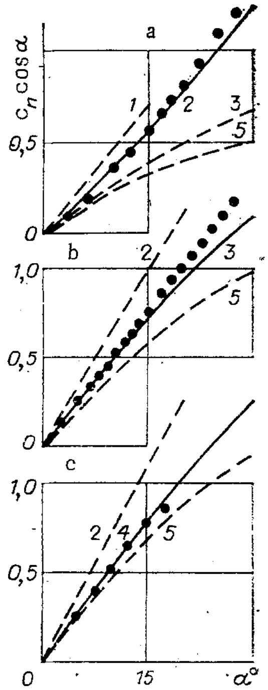

Fig. 5

Formula (6) sets up a connection between the intensity of the vortex sheet convergent with the leading edge and the sweep angle of this wing $\chi = (\pi/2) - \theta$ for delta wings. In the case of a plane wing with a curvilinear edge the intensity of the vortex sheet convergent with the leading edge will vary depending on the local slope of the edge θ according to the formula (6).

Results of computing the lift coefficients $c_y = c_n \cos\alpha$ for thin plane rectangular wings of different span λ are presented in Fig. 4 as a function of the angle of attack α for the scheme of unseparated flow around the leading and side edges ($K_l = K_s = 0$) (dashed lines) as well as for the scheme of unseparated flow around the leading edge ($\theta_l = \pi/2$, $K_l = 0$) and total separation at the side edges ($\theta_s = 0$, $K_s = 1$) corresponding to selection of the factor K by means of (7) (solid lines). In both cases the usual condition of smooth convergence of the sheet and build-up of the sheet at infinite in the direction of the unperturbed free stream velocity is satisfied at the trailing edge. Experimental results [7] for thin ($\bar{c}$ = 1.2%) rectangular plates of appropriate span are superposed by points in Fig. 4. It is seen that the results of computing the aerodynamic characteristics with a convergent sheet of intensity K selected in conformity with (7) taken into account, are in satisfactory agreement with experimental data down to angles of attack at which the flow mode changes, a stall in the flow appears at the leading edge ($\alpha \approx 30°$ for $\lambda = 0.5$, $\alpha \approx 25°$ for $\lambda = 1$, $\alpha \approx 15°$ for $\lambda = 2$, in Fig. 4a-c, respectively).

Shown in Fig. 5 are results of computing the lift coefficient $c_y = c_n \cos\alpha$ for thin delta wings of the span: a) $\lambda = 1$ ($\theta = 14°$), b) $\lambda = 2.3$ ($\theta = 30°$), c) $\lambda = 3$ ($\theta = 37°$) as a function of the angle of attack for K = 1, 0.75, 0.5, 0.4, 0 in lines 1-5. The solid lines correspond to computations with a value of the parameter K selected according to (7). To show the degree of influence of the parameter K on the computations, results are presented by dashes for other values of the parameter K. Experimental results [7] (see [2], also) are superposed by points in Fig. 5 for thin ($\bar{c}$ = 1.2%) delta plates of appropriate spans. It is seen that the results of computations taking account of the vortex sheet of intensity K, determined from (7), convergent with the leading edge, are in satisfactory agreement with experimental data for the angle of attack $\alpha \leqslant 15°$.

The values of the parameter K whose confidence is confirmed by comparing the computed total aerodynamic characteristics with data of experiment for thin rectangular and delta plates up to the angles of attack $\alpha \leqslant 15°$ (see Figs. 4 and 5) are superposed by circle in the graph of Fig. 3 obtained from (7).

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HEAT TRANSFER IN LAVAL NOZZLES WHEN A SCREEN IS PRESENT

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We present the results of an investigation of heat exchange when a screen is present and under complicated flow conditions: an accelerated, compressible, axisymmetric stream with compression shocks.

The tests were made on an exerimental installation described in detail in [1]. The working sections of the installation were interchangeable, supersonic, conical nozzles. The subcritical part of the nozzles had the same geometry: diameters of entrance and critical cross sections 80 and 20 mm, convergence half-angle $\varphi_1/2 = 30°$. The supersonic parts of the nozzles differed in the expansion angles, which had values of $\varphi_2/2 = 6, 30, 40°$. To measure the wall temperature, thermocouples 0.2 mm in diameter were imbedded flush with the inner surface along a generating line of the nozzle. Openings 0.4 mm in diameter for measuring the static pressure were drilled in the same cross sections where the thermocouples were mounted.

In the tests the heat flux was directed from the wall to the main air stream, having a stagnation temperature $T_0 \approx 288°K$. To increase the accuracy of the experimental determination of the heat-exchange coefficient, we developed a special method of wall heating [2], aimed at considerably reducing the heat leaks. Its essence consists of the following. A graphite-based liquid mixture was deposited in a uniform layer onto a section, consisting of a strip with a constant width of 30 mm, of the inner surface of the textolite nozzle. After drying, a thin electrically conducting film $\sim$40 μm thick was formed on the wall. It was heated by passing an electric current through it, and the amount of heat released was determined from the measured power. The heat-flux density was found from the ratio of this heat to the area of the film. Since the film had a constant width and uniform heat release, the heat-flux density was constant along its length. The uniformity of heat release was monitored by the constancy of the electric resistance of individual sections of the film and by equality of the film temperature when it was heated by a current under conditions of the absence of convective heat exchange.

Because of the low thermal conductivity of the nozzle wall and of the thin electrically heated layer, the longitudinal heat leaks are small, which is especially important under the conditions of large temperature gradients in the direction of the x axis which occurred in

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